Homework Sections 1.6-2.2

Max's lecture
MATH 55

Due July 2, 2019

Note: All problems are taken from Rosen, Discrete Mathematics and its applications, 8th ed. Have fun, and please feel free to ask each other and me for help!

Exercise 1.6.4. Which rule of inference is used in each of these arguments?

(a) Kangaroos live in Australia and are marsupials. Therefore, kangeroos are marsupials.

(b) It is either hotter than 100 degrees today or the pollution is dangerous. It is less than 100 degrees outside. Therefore, the pollution is dangerous.

(c) Linda is an excellent swimmer. If Linda is an excellent swimmer, then she can work as a lifeguard. Therefore, Linda can work as a lifegaurd.

(d) Steve will work at a computer company this summer. Therefore, this summer Steve will work at a computer company or he will be a beach bum.

(e) If I work all night on this homework, then I can answer all the exercises. If I answer all the exercises, I will understand the material. Therefore, if I work all night on the homework, then I will understand the material.

Exercise 1.6.6. Use rules of inference to show that the hypothesis "If it does not rain or if it is not foggy, then the sailing race will be held and the lifesaving demonstratoin will go on", "If the sailing race is held, then the trophy will be awarded", and "The trophy was not award", imply the conclusion "It rained".

Exercise 1.6.8. What rules of inference are used in this argument. "No man is an island. Manhattan is an island. Therefore, Manhattan is not a man."

Exercise 1.6.16. For each of these arguments determine whether the argument is correct or incorrect and explain why.

(a) Eveyone enrolled in the university has lived in a dormitory. Mia has never lived in a dormitory. Therefore, Mia is not enrolled in the university.

(b) A convertible car is fun to drive. Isaac's car is not a convertible. Therefore, Isaac's car is not fun to drive.

(c) Quincy likes all action movies. Quincy likes the movie Eight Men Out. Therefore, Eight Men Out is an action movie.

(d) All lobstermen set at least a dozen traps. Hamilton is a lobsterman. Therefore Hamilton sets at least a dozen traps.

Exercise 1.7.8. Prove that if n is a perfect square, then $n+2$ is not a perfect square.

Exercise 1.7.10. Use a direct proof to show that the product of two rational numbers is rational.

Exercise 1.7.11. Prove or disprove that the product of two irrational numbers is irrational.

Exercise 1.7.16. Prove that if x, y, z are integers and $x+y+z$ is odd, then at least one of x, y, z is odd.

Exercise 1.7.18. Prvoe that if m and n are integers and mn is even, then m is even or n is even.

Exercise 1.7.38. Show that the propositions p_1, p_2, p_3, p_4 can be shown to be equivalent by showing that $p_1 \leftrightarrow p_4$, $p_2 \leftrightarrow p_3$, and $p_1 \leftrightarrow p_3$.

Exercise 1.8.2. Use a proof by cases to show that 10 is not the square of a positive integer. [Hint: consider two cases, $1 \leq x \leq 3$ and $x \geq 4$.]

Exercise 1.8.8. Prove using the notion of without loss of generality that $5x+5y$ is an odd integer when x an y are integers of the opposite parity.

Exercise 1.8.12. Prove that either $2 \cdot 10^{500} + 15$ or $2 \cdot 10^{500} + 16$ is not a perfect square. Is your proof constructive or nonconstructive?

Exercise 1.8.18. Show that a, b, c are real numbers and $a \neq 0$, there exists a unique solution to the equation $ax + b = c$.

Exercise 1.8.46. Prove or disprove that you can use dominoes to tile a 5×5 checkerboard with 3 corners removed.

Exercise 2.1.12. Determine whether these statements are true or false.

(a) $\emptyset \in \{\emptyset\}$

(b) $\emptyset \in \{\emptyset, \{\emptyset\}\}$

(c) $\{\emptyset\} \in \{\emptyset\}$

(d) $\{\emptyset\} \in \{\{\emptyset\}\}$

(e) $\{\emptyset\} \subset \{\emptyset, \{\emptyset\}\}$

(f) $\{\{\emptyset\}\} \subset \{\emptyset, \{\emptyset\}\}$

(g) $\{\{\emptyset\}\} \subset \{\{\emptyset\}, \{\emptyset\}\}$

Exercise 2.1.22. What is the cardinality of each of these sets?

(a) $\emptyset$

(b) $\{\emptyset\}$

(c) $\{\emptyset, \{\emptyset\}\}$

(d) $\{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}$

Exercise 2.1.29. Let $A = \{a, b, c, d\}$ and let $B = \{y, z\}$. Find $A \times B$ and $B \times A$.

Exercise 2.1.38. How many different elements does $A \times B \times C$ have if A has m elements, B has n elements, and C has p elements.

Exercise 2.2.4. Let $A = \{a, b, c, d, e\}$ and $B = \{a, b, c, d, e, f, g, h\}$. Find:

(a) $A \cup B$

(b) $A \cap B$

(c) $A - B$

(d) $B - A$

Exercise 2.2.16e. Let A and B be sets. Show that $A \cup (B - A) = A \cup B$

Exercise 2.2.18. Given sets A and B in universe U, draw the Venn diagrams of each of these sets.

(a) $A \to B = \{x \in U | x \in A \to x \in B\}$

(b) $A \leftrightarrow B = \{x \in U | x \in A \leftrightarrow x \in B\}$

Exercise 2.2.22. Show that if A and B are sets with $A \subseteq B$, then $A \cup B = B$ and $A \cap B = A$.